Math 2FM3, Tutorial 1

Sep 16th , 2015

TA Information

- Name: Chengwei Qin (PhD candidate in Stats)
- E-Mail: qinc4@math.mcmaster.ca
- Webpage: http://ms.mcmaster.ca/~qinc4
- Office hours: 10am to 12pm, Friday, HH401

Preliminary

- Compound Interest:

  $A(t)=A(0)(1+i)^t$

- Simple Interest:

  $A(t)=A(0)(1+it)$

- Present Value:

  $A(0)=A(t)v^t$, $v=(1+i)^{-1}$ is present value factor

Ex 1.1.2

- 2500 is invested. Find the accumulated value of the investment 10 years after it is made for each of the following rates:

  (a) 4% annual simple interest;

  (b) 4% effective annual compound interest;

  (c) 6-month interest rate of 2% compounded every 6 months;

  (d) 3-month interest rate of 1% compounded every 3 months.

(a) $A(0)=2500$, $t=10$, $i=4\%$

$A(t)=A(0)(1+it)=2500*(1+4\%*10)=3500.$

(b) $A(t)=A(0)(1+i)^t = 2500*(1+4\%)^{10} = 3700.61$

(c) $A(0)=2500$, $t=10/0.5=20$, $i=2\%$

$A(t)=A(0)(1+i)^t =2500*(1+2\%)^{20} =3714.87$

(d) $A(0)=2500$, $t=10/0.25=40$, $i=1\%$

$A(t)=A(0)(1+i)^t =2500*(1+1\%)^{40} =3722.16$

Ex 1.1.10

- (a) At an effective annual interest rate of 12%, calculate the number of years (including fractions) it will take for an investment of 1000 to accumulate to 3000.
- (b) Repeat part (a) using the assumption that for fractions of a year, simple interest is applied.
- (c) Repeat part (a) using an effective monthly interest rate of 1%.
- (d) Suppose that an investment of 1000 accumulated to 3000 in exactly 10 years at effective annual rate of interest i. Calculate i.
- (e) Repeat part (d) using an effective monthly rate of interest j. Calculate j.

(a) $A(0)=1000$, $A(t)=3000$, $i=12\%$

Since $(1+i)^t = A(t)/A(0)$,

take "ln" we have

$t*\ln(1+i)=\ln A(t)-\ln A(0)$,

then $t=(\ln A(t)-\ln A(0))/\ln(1+i)=9.694$

(b) The integer part is 9, then we use compound interest for the first 9 years and simple interest for the last year.

$A(t)=A(0)(1+i)^9 (1+i*s)=3000$, $s=0.6819$

$t=9+0.6819=0.6819$

(c) $i=0.01$,

$$1000(1+0.01)^t=3000$$

$t=110.41$ months

(d) $(1+i)^t = A(t)/A(0)$

$$i=(A(t)/A(0))^{1/t} -1= (3000/1000)^{1/10} -1=0.1161$$

(e)

$$j=(A(t)/A(0))^{1/t} -1= (3000/1000)^{1/120} -1=0.009197$$

Ex 1.2.1

- Bill will receive $5000 at the end of each year for the next 4 years. Using an effective annual interest rate of 6%, find today’s present value of all the payments Bill will receive.

- $v=1/(1+i)=1/(1+6\%)=0.9434$

  present value=$5000v+5000v^2 +5000v^3+ 5000v^4=17,325.53$

Ex 1.2.4

- What is the present value of 1000 due in 10 years if the effective annual interest rate is 6% for each of the first 3 years, 7% for the next 4 years and 9% for the final 3 years?

First 3 years : $v1=1/(1+6\%)=0.9434$

Next 4 years: $v2=1/(1+7\%)=0.9346$

Final 3 years: $v3=1/(1+9\%)=0.9174$

Then the present value is

$1000(v1)^3 (v2)^4 (v3)^3 = 494.62$